Workshop on
Computational Commutative Algebra, July 2-7, 2011, IPM, Tehran, Iran

Computational Homological Methods in Commutative Algebra

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Homological algebra is a vast generalization of linear algebra and, as such, an indispensable tool with deep applications to numerous ares of mathematics, such as algebraic topology, group cohomology, representation theory, commutative algebra, algebraic geometry, noncommutative algebra, D-module theory, system and control theory, etc.

Constructive homological algebra is a modern discipline which combines the strength of the abstract homological notions with the increasing power of modern computers.

In this series of lectures I will start by introducing the notion of a computable Abelian category, which is, in my opinion, the shortest path to constructive homological algebra. All abstract homological constructions become effectively computable over such categories. The most prominent computable Abelian category is the category of finitely presented modules over a computable ring.

I will try to touch upon the following topics from a constructive point of view:

- Basic homological algebra in Abelian categories: kernels, cokernels, images, projective resolutions, the snake Lemma, and long exact sequences
- The functors Ext and Tor
- Koszul complexes and Ext
- Flatness, projectivity, Fitting ideals, and Tor
- The category of finitely presented graded modules, Betti diagrams, Hilbert-Poincare polynomials, Castelnuovo-Mumford regularity
- Local cohomology and the higher sheaf cohomology modules